Abstract

- We verify the metrological usefulness of quantum states based on a few expectation values.
- We estimate the quantum Fisher information connected with the metrological usefulness.
- Our approach is optimal. It gives a tight bound from below for the QFI.
- The archetypical bound for the QFI [1]:

\[ F_Q(\rho, \mathcal{J}) \geq \frac{\langle \mathcal{J} \rangle^2}{\langle \Delta \mathcal{J} \rangle^2} \]  

(1)

The bound from below

Legendre transform for convex entanglement measures,

\[ g(rW) = \sup_{\rho} \{ rW, g(\rho) \}. \]

If convex-roof [2],

\[ g(rW) = \sup_{\psi} \{ rW, g(\psi) \}. \]

The QFI is a convex-roof over the states [3]. Thus, the bound based on expectation values is

\[ B_Q(\psi_1, \psi_2, \ldots) = \sup_r \{ rW - \sup_{\psi} \{ rW - 4\langle \mathcal{J} \rangle^2 \} \}. \]

The inner maximisation is simplified,

\[ F_Q(rW) = \sup_{\rho} \{ rW - 4\langle \mathcal{J} \rangle^2 \}. \]

Fidelity based bounds

For GHZ fidelity,

\[ F_Q = \left\{ \begin{array}{ll} 1 - 2F_{GHZ}^2 , & F_{GHZ} \geq 0.5 \\ 0, & F_{GHZ} < 0.5 \end{array} \right. \]

Analytical solution!

The unpolarized Dicke state is defined as

\[ |\psi_{NP}^N\rangle = \left( \frac{N}{N!} \right)^{1/2} \sum_{\pi} \prod_n (|n\rangle^{\pi_n} \otimes |n\rangle^{\pi_n}), \]

where \( \pi_n \) are the \( N! \) permutations.

Spin-squeezing parameters

Set of observables: \( \langle \mathcal{J}_+ \rangle, \langle \mathcal{J}_- \rangle^2 \)

- (upper-right) QFI bound for 6-particle system scanned for all physically allowed values. (dashed) Threshold for over classical metrology. We see agreement with respect to Equation (1) on the bottom half.
- (middle-right) Comparison between Eq. (1) and the optimal bound just on the lower boundary of the physically allowed. (dark to light) Relative difference for 4, 6, 10, 20 and 1000 particles.
- (Lower) Bounds computed for states allowed by bosonic symmetry for a given ratio of \( \frac{N}{N+1} \).

Unpolarised Dicke states

Set of observables with constraint:

\[ \langle \mathcal{J}_+ \rangle \text{ and } \langle \mathcal{J}_- \rangle = \langle \mathcal{J}_x \rangle \]

- (upper) QFI bound for 6-particle system for all possible values. Bosonic symmetry is imposed,

\[ \langle \mathcal{J}_+ \rangle + \langle \mathcal{J}_- \rangle = \frac{N(N+2)}{4} \]

- (lower) Bound for experimental data [4]. Even though, the system is a 7900-particle system, scaling techniques were used to asymptotically obtain the bound.

\[ \langle \mathcal{J}_+ \rangle = 112 \pm 31 \]

\[ \langle \mathcal{J}_- \rangle = \langle \mathcal{J}_x \rangle = |6 \pm 0.631| \times 10^6 \]

Conclusions

We tested our approach even for large particle number systems. In the future, it would be interesting to use our method to test the optimality of different formulas or improve existing bounds with more observables.

Bibliography and related material


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